Anomalous dimensions of gauge-invariant amplitudes in massless effective gauge theories of strongly correlated systems

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We use the radial gauge to calculate the recently proposed ansatz for the physical electron propagator in such effective models of strongly correlated electron systems as the QED_3 theory of the pseudogap phase of the cuprates. The results of our analysis help to settle the recent dispute about the sign and the magnitude of the anomalous dimension which characterizes the gauge invariant amplitude in question and set the stage for computing other, more physically relevant, ones.

I. INTRODUCTION

As a generic property, the one-dimensional Fermi systems with short-range (screened) repulsive interactions routinely demonstrate algebraic decay of all the correlation functions governed by non-universal (coupling-dependent) anomalous exponents.

A possibility of the emergence of a similar behavior, commonly referred to as the "Luttinger liquid", in higher dimensional strongly correlated electron systems has been extensively discussed in the recent literature.

Thus far, however, no solid consensus has been reached even on the necessary criteria that have to be fulfilled for the Luttinger behavior to set in, much less on whether or not it occurs in any specific example of a strongly correlated electron system. It was largely for this reason that the attention has recently been drawn to the class of effective models described by (possibly, spatially anisotropic and/or Lorentz non-invariant) deformations of the standard action of Quantum Electrodynamics.

Motivated by the puzzling properties of the quasi-two-dimensional high-temperature copper-oxide superconductors, most of the interest has been focused on the three-dimensional case described by either the ordinary (parity-even) QED_3 or the abelian 3D Chern-Simons theory[1,2], where the finite density problem of non-relativistic (massive) fermions has become the main subject of the scrutiny. However, the latter was found to fall into a rather different class of non-Fermi liquids which bear little resemblance to the 1D Luttinger liquid[3].

Recently, the idea of the conjectured Luttinger-like behavior has been rekindled in the recent theories of the pseudogap phase of the underdoped superconducting cuprates[4–8]. Albeit describing rather different physics, all these approaches resort to the same effective description in terms of the pseudo-relativistic QED_3 theory of the gapless nodal fermion excitations which retain their d-wave symmetrical gap well above the critical temperature T_c regarded merely as the onset of global phase coherence.

Above T_c the fermionic excitations experience strong scattering by both thermal and quantum fluctuations of an incipient ordering, such as a flux of the gauge field measuring a local spin chirality[4,5] or vortex-antivortex pairs of the pairing order parameter[6–8]. The latter scenario has recently received a new experimental support from the observation that the vortex matter is present at temperatures well in excess of T_c, as revealed by the measurements of the Nernst effect[9].

As another important ingredient, the QED_3 theory of the pseudogap phase was aimed at explaining the ubiquitous destruction of the coherent quasiparticles above T_c which was observed in angular-resolved photoemission and tunneling experiments.

To this end, the authors of Ref.[5] conjectured that the electron propagator in question may, in fact, exhibit the Luttinger behavior

$$G(x) \propto \hat{x}/|x|^{3+\eta}, \quad \hat{x} = \gamma_\mu x_\mu, \tag{1}$$

characterized by a positive anomalous exponent $\eta > 0$, and they also attempted to fit the ARPES data, while claiming a good agreement with experiment (unless explicitly stated otherwise, throughout this paper we use the notation $\hat{n} \equiv \gamma_\mu n_\mu$ for vectors n_μ contracted with the Dirac matrices γ_μ).

The conclusions drawn in Ref.[5] were based on the use of the following heuristic form of the gauge-invariant electron propagator

$$G(x-y) = \langle 0|\psi(x) \exp\left[-i\int_\Gamma dz^\mu\, A_\mu(z)\right]\bar{\psi}(y)|0\rangle, \tag{2}$$

where the line integral was taken along the contour Γ chosen as the straight-line segment connecting the end points.

Later on, the calculations of Ref.[5] were carried out by a number of other authors, and the results for the anomalous exponent appeared to vary not only between the different authors ($\eta = 32/3\pi^2 N$ [5] vs $-32/3\pi^2 N$ [10]) but also from one to another work of the same authors ($\eta = -16/3\pi^2 N$ [6] vs $16/3\pi^2 N$ [11] and $\eta = 32/3\pi^2 N$ [7] vs $-64/3\pi^2 N$ [12]).

While some of the calculations were performed in the conventional covariant gauges[10,12], other authors made use of the potentially problematic axial gauge ($(x-y)_\mu A_\mu(z) = 0$ where x, y are arbitrarily chosen points which are taken to coincide with the end points of the contour Γ)[5,6,10] which spurred a debate over the issue of a true (vs limited, see[13]) gauge invariance of Eq. (2), as opposed to its surrogate functions proposed in[11] (see Summary for a more extended discussion).

While seemingly being an issue of secondary importance, a proper construction of the physical electron propagator is, in fact, imperative, as far as ascertaining the status of the conjectured Luttinger-like behavior in the QED-like theories is concerned.

In light of the present controversy, in this paper we undertake yet another attempt to settle the dispute about the physically motivated form of the electron propagator and the actual value of η (if any) by resorting to the so-called radial (Fock-Schwinger) gauge ($(z-x)_\mu A_\mu(z) = 0$ and x is an arbitrary fixed point). The radial gauge is known to be free of the potential problems that might exist in the axial gauge, which, according to some authors, may even require one to introduce ghost fields[14]. In addition, we also set out to explore the dependence of the previously conjectured form of the electron propagator (2) on the choice of the contour Γ.

II. GAUGE INVARIANT FERMION PROPAGATOR IN THE FOCK-SCHWINGER GAUGE

We start with the $3D$ relativistic theory of massless Dirac spinors coupled to a massless $U(1)$ gauge field, whose Euclidean action is

$$S\left[\psi, \bar{\psi}, A\right] = \int d^3x \sum_{i=1}^{N} \bar{\psi}_i \gamma_\mu \left(\partial_\mu - iA_\mu\right) \psi_i, \tag{3}$$

where $\bar{\psi} \equiv \psi^\dagger \gamma_0$ and the N-flavored Dirac fermions are described by four-component bi-spinors which belong to a reducible representation of the γ-matrices satisfying the Clifford algebra $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$ ($\mu, \nu = 0, 1, 2$). The latter can be chosen in the form of the direct product $\gamma_\mu = \sigma_3 \otimes (\sigma_3, \sigma_2, \sigma_1)$ of the standard triplet of the Pauli matrices σ_μ.

In all of the above mentioned condensed matter-inspired QED-like models the number of fermion flavors $N = 2$. Nevertheless, in what follows we choose to treat N as a parameter that can assume arbitrary values, depending on the problem in question.

The dynamics of the $U(1)$ gauge field is generated by the effective action obtained after tracing out the fermionic degrees of freedom

$$S_{\text{eff}}[A] = \frac{1}{2} \int d^n x \int d^n y A_\mu(x) D^{-1}_{\mu\nu}(x-y) A_\nu(y) + \ldots, \tag{4}$$

where the dots stand for the higher order terms (non-Gaussian) which we hereafter neglectm as is done in all of the previous works on the subject. Intending to subsequently use the method of dimensional regularization for evaluating Feynman diagrams we formulated Eq.4 in $n = 3 - \epsilon$ dimensions. Also, in Eq.4 we neglected the bare Maxwell term $\sim (\partial_\mu A_\nu - \partial_\nu A_\mu)^2$ which turns out to be irrelevant in the low-energy, long-distance limit.

The previously proposed candidate for the physical electron propagator (2) studied in[5–7,10–12] can be cast in the following form

$$G_{inv}(x-y) = \langle G[x, y; A] \exp\left[-i \int\limits_y^x dz^\mu \, A_\mu(z) \right] \rangle, \tag{5}$$

where $G[x, y; A]$ is a fermion propagator for a given fixed configuration of the gauge field A_μ, and the brackets stand for the (normalized) functional average over the gauge field which is described by the action Eq.(4).

In the Euclidean momentum space, the kernel $D^{-1}_{\mu\nu}$ of the quadratic operator has the form

$$D^{-1}_{\mu\nu}(q) = \frac{N\sqrt{q^2}}{8} \left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right). \tag{6}$$

Introducing a source field

$$J^\mu(z) = (x-y)^\mu \int\limits_0^1 d\alpha \, \delta^n(z - y - (x-y)\alpha), \tag{7}$$

we can write the straight-line integral which appears in Eq. (2) as

$$\int\limits_y^x dz^\mu \, A_\mu(z) = \int d^n z \, J^\mu(z) A^\mu(z). \tag{8}$$

In the Fock-Schwinger gauge

$$(x - x_0)_\mu A_\mu(x) = 0 \tag{9}$$

the line integral in Eq. (5) vanishes if one chooses the reference point x_0 at the "center of mass" $x_0 = X = (x+y)/2$ (for the proof, see, Appendix A which also contains a derivation of the photon propagator in the FS gauge $D^{FS}_{\mu\nu}(z_1 + X, z_2 + X)$).

We compute the first order $1/N$ correction to the fermion propagator (3) by expanding the inverse Dirac operator $1/(\hat{\partial}-i\hat{A}) = 1/\hat{\partial} + (1/\hat{\partial})(i\hat{A})(1/\hat{\partial}) + \ldots$ in powers of $A_\mu(z)$, thus obtaining

$$G_{inv}(x,y) = S(x-y) + \delta G(x,y), \qquad (10)$$

where $S(x-y)$ and $\delta G(x,y)$ are the bare fermion propagator and the correction to it, respectively. The latter is given by the expression

$$\delta G(x,y) = -\int d^n z_1 \int d^n z_2\, S(x-z_1-X)\gamma^\mu \times S(z_1-z_2)\gamma^\nu S(z_2+X-y) D^{FS}_{\mu\nu}(z_1+X, z_2+X), \qquad (11)$$

where it is natural to decompose the propagator $D^{FS}_{\mu\nu}$ for the gauge fields in the Fock-Schwinger gauge into four contributions,

$$D^{FS}_{\mu\nu}(x,y) = \sum_{i=1}^{4} D^{(i)}_{\mu\nu}(x,y), \qquad (12)$$

as is done explicitly in Eqs. (A20) to (A23). By choosing the reference point in the Fock-Schwinger gauge to be the center of mass X, one verifies by inspection of Eqs. (A20) to (A23) that the contribution δG can only depend on the relative coordinate $\bar{x} = x - y$:

$$\delta G(\bar{x}) = -\int d^n z_1 \int d^n z_2\, S(\bar{x}/2 - z_1)\gamma^\mu \times S(z_1-z_2)\gamma^\nu S(z_2+\bar{x}/2) D^{FS}_{\mu\nu}(z_1+X, z_2+X). \qquad (13)$$

Taking the Fourier transform of Eq. (13) $FT[\delta G(\bar{x})] = i\delta G(p)$, $FT[S(\bar{x})] = iS(p)$, $S(p) = 1/\hat{p}$, we obtain

$$\delta G^{(i)}(p) = \int \frac{d^n q_1}{(2\pi)^n} \frac{d^n q_2}{(2\pi)^n} S(p + q_1/2 + q_2/2)\gamma^\mu \times S(p - q_1/2 + q_2/2)\gamma^\nu S(p - q_1/2 - q_2/2) D^{(i)}_{\mu\nu}(q_1, q_2). \qquad (14)$$

In the above equation, the index $i = 1, \ldots, 4$ labels the Fourier transforms $D^{(i)}_{\mu\nu}(q_1,q_2)$ of the components $D^{(i)}_{\mu\nu}(z_1+X, z_2+X)$ given by Eqs.(A20)-(A23)

$$D^{(1)}_{\mu\nu}(q_1,q_2) = (2\pi)^n \delta^n(q_1+q_2)\delta_{\mu\nu} D(q_1), \qquad (15)$$

$$D^{(2)}_{\mu\nu}(q_1,q_2) = \int_0^1 d\alpha\, q_{1\mu} \frac{\partial}{\partial q_{1\nu}} D^{(\alpha)}(q_1,q_2), \qquad (16)$$

$$D^{(3)}_{\mu\nu}(q_1,q_2) = \int_0^1 d\beta\, q_{2\nu} \frac{\partial}{\partial q_{2\mu}} D^{(\beta)}(q_2,q_1), \qquad (17)$$

$$D^{(4)}_{\mu\nu}(q_1,q_2) = \int_0^1 d\alpha \int_0^1 d\beta q_{1\mu} q_{2\nu} \frac{\partial}{\partial q_{1\lambda}} \frac{\partial}{\partial q_{2\lambda}} D^{(\alpha,\beta)}(q_1,q_2), \qquad (18)$$

where

$$D(q) = \frac{8}{N}\frac{1}{\sqrt{q^2}}, \qquad (19)$$

$$D^{(\alpha)}(q_1,q_2) = (2\pi)^n \delta^n(q_1 + \alpha q_2) D(q_1/\alpha), \qquad (20)$$

$$D^{(\alpha,\beta)}(q_1,q_2) = (2\pi)^n \delta^n(\beta q_1 + \alpha q_2) D(q_1/\alpha). \qquad (21)$$

The first contribution to the anomalous dimension stems from $\delta G^{(1)}(p)$ given by Eqs. (14) and (15). Since the Fourier transform of the propagator Eq.(1) is proportional to $\hat{p}/p^{2-\eta}$ it proves convenient to multiply $\delta G^{(1)}(p)$ by $\hat{p}$ in order to deduce the exponent η which we are after. Now taking the trace we arrive at the following result

$$\frac{1}{4}\mathrm{Tr}\left[\hat{p}\delta G^{(1)}(p)\right] = \frac{1}{4}\int \frac{d^n q}{(2\pi)^n} \mathrm{Tr}[\hat{p} S(p)\gamma^\mu S(p-q)\gamma^\mu \times S(p)] D(q) = -\frac{8(n-2)}{N}\int \frac{d^n q}{(2\pi)^n} \frac{p^2 - p\cdot q}{\sqrt{q^2}p^2(p-q)^2} = -\frac{4}{3\pi^2 N}\frac{|p|^{-\epsilon}}{\epsilon}, \qquad (22)$$

where we used dimensional regularization near $n = 3-\epsilon$ in the divergent integral

$$\int \frac{d^n q}{(2\pi)^n} \frac{p^2 - p\cdot q}{\sqrt{q^2}p^2(p-q)^2} = \frac{1}{6\pi^2}\frac{|p|^{-\epsilon}}{\epsilon}. \qquad (23)$$

Notably, Eq. (22) coincides with the result for the anomalous dimension of the ordinary (gauge-variant) fermion propagator performed in the covariant Feynman gauge.

Next, we compute the term $\delta G^{(2)}(p)$ given by Eqs. (14) and (16))

$$\delta G^{(2)}(p) = \int_0^1 d\alpha \int \frac{d^n q_1}{(2\pi)^n}\frac{d^n q_2}{(2\pi)^n} S(p + q_1/2 + q_2/2) \times \gamma^\mu S(p - q_1/2 + q_2/2)\gamma^\nu S(p - q_1/2 - q_2/2) \times q_{1\mu}\frac{\partial}{\partial q_{1\nu}}(2\pi)^n \delta^n(q_1 + \alpha q_2) D(q_1/\alpha). \qquad (24)$$

Integrating by parts we cast $\delta G^{(2)}(p)$ in the form

$$\delta G^{(2)}(p) = -\int_0^1 d\alpha \int \frac{d^n q_1}{(2\pi)^n}\frac{d^n q_2}{(2\pi)^n}(2\pi)^n \delta^n(q_1 + q_2\alpha) \times D(\frac{q_1}{\alpha})\frac{\partial}{\partial q_{1\nu}}\left[\left[S(p - \frac{q_1}{2} + \frac{q_2}{2}) - S(p + \frac{q_1}{2} + \frac{q_2}{2})\right] \times \gamma^\nu S(p - \frac{q_1}{2} - \frac{q_2}{2})\right], \qquad (25)$$

where we made use of the Ward-Takahashi identity (WTI) for the bare propagators

$$S(k+q)\hat{q}S(k) = S(k) - S(k+q). \tag{26}$$

Multiplying Eq. (25) by $\hat{p}$ and taking the trace we obtain

$$\frac{1}{4}\mathrm{Tr}\left[\hat{p}\delta G^{(2)}(p)\right] = -\int_0^1 d\alpha \int \frac{d^n q_1}{(2\pi)^n}\frac{d^n q_2}{(2\pi)^n}$$
$$\times(2\pi)^n\delta^n(q_1+q_2\alpha)D(q_1/\alpha)$$
$$\times\Bigg\{-\frac{(n-2)}{2}\frac{(2p^2-p\cdot q_1)}{(p-q_1/2+q_2/2)^2(p-q_1/2-q_2/2)^2}$$
$$+\frac{(n-2)}{2}\frac{(p\cdot q_1+p\cdot q_2)}{(p-q_1/2-q_2/2)^2(p+q_1/2+q_2/2)^2}\Bigg\}. \tag{27}$$

Being an odd function of the momenta $q_{1,2}$ the last term in Eq. (27) vanishes. Thus, we obtain

$$\frac{1}{4}\mathrm{Tr}\left[\hat{p}\delta G^{(2)}(p)\right] = \frac{4(n-2)2^n}{N}\int_0^1 d\alpha \int \frac{d^n q}{(2\pi)^n}$$
$$\times\frac{(p^2+p\cdot q\alpha)}{\sqrt{q^2}(p+q(1+\alpha))^2(p-q(1-\alpha))^2}. \tag{28}$$

The momentum integral can be evaluated by virtue of the Feynman parameterization

$$I_1 = \int \frac{d^n q}{(2\pi)^n}\left[\frac{(p^2+p\cdot q\alpha)}{\sqrt{q^2}(p+q(1+\alpha))^2(p-q(1-\alpha))^2}\right]$$
$$= \frac{p^2}{(1-\alpha^2)^3}\frac{3}{4}\int_0^1 dx\int_0^1 dy\,\frac{x}{\sqrt{1-x}}(1-(2xy-x)\alpha$$
$$-(1-x)\alpha^2)\int_0^\infty \frac{dq}{2\pi^2}\frac{q^{n-1}}{[q^2+c]^{5/2}}$$
$$= \frac{p^2}{(1-\alpha^2)^3}\frac{3}{4}\int_0^1 dx\int_0^1 dy\,\frac{x}{\sqrt{1-x}}(1-(2xy-x)\alpha$$
$$-(1-x)\alpha^2)\frac{c^{(n-5)/2}}{6\pi^2}, \tag{29}$$

where the argument c takes the form

$$c = \frac{p^2 x}{(1-\alpha^2)^2}\left[1-x(1-2y)^2+2(1-x)(1-2y)\alpha\right.$$
$$\left.+\,(1-x)\alpha^2\right]. \tag{30}$$

The leading divergence of the integral over α can be extracted by making an approximation similar to that of Ref.[16]

$$c^{(n-5)/2} \approx \left[\frac{p^2}{(1-\alpha^2)^2}\right]^{(n-5)/2}$$
$$\times\frac{1}{x\left[1-x(1-2y)^2+2(1-x)(1-2y)\alpha+(1-x)\alpha^2\right]}. \tag{31}$$

In this way, we obtain

$$I_1 \approx \frac{1}{8\pi^2(1-\alpha^2)}\left[\frac{|p|}{(1-\alpha^2)}\right]^{-\epsilon} I(\alpha), \tag{32}$$

where

$$I(\alpha) = \int_0^1 dx\int_0^1 dy$$
$$\times\frac{(1-x)^{-1/2}(1-(2xy-x)\alpha-(1-x)\alpha^2)}{[1-x(1-2y)^2+2(1-x)(1-2y)\alpha+(1-x)\alpha^2]}. \tag{33}$$

The expression for $\delta G^{(2)}$ now reads

$$\frac{1}{4}\mathrm{Tr}\left[\hat{p}\delta G^{(2)}(p)\right] = \frac{4|p|^{-\epsilon}}{\pi^2 N}\int_0^1 d\alpha\,\frac{I(\alpha)}{(1-\alpha^2)^{1-\epsilon}}$$
$$\approx I(1)\int_0^1 d\alpha\frac{1}{2(1-\alpha)^{1-\epsilon}} = \frac{4}{N\pi^2}\frac{|p|^{-\epsilon}}{\epsilon}, \tag{34}$$

where we used the integral

$$I(1) = \int_0^1 dx\int_0^1 dy\,\frac{x(1-x)^{-1/2}}{2[1+x(y-1)]} = 2. \tag{35}$$

It can be readily shown that Eq. (34) is also identical to the result for $\mathrm{Tr}[\hat{p}\delta G^{(3)}(p)]$, given by Eqs. (14) and (17).

Lastly, the expression for $\delta G^{(4)}(p)$ given by Eqs. (14) and (18) reads as

$$\delta G^{(4)}(p) = \int_0^1 \frac{d\alpha}{\alpha^{n-2}}\int_0^{1/\alpha} d\tau\int\frac{d^n q_1}{(2\pi)^n}\frac{d^n q_2}{(2\pi)^n}$$
$$\times(2\pi)^n\delta^n(q_1\tau+q_2)D(q_1)\frac{\partial}{\partial q_1^\lambda}\frac{\partial}{\partial q_2^\lambda}\Big[S(p+\frac{q_1}{2}+\frac{q_2}{2})$$
$$\times\,\hat{q}_1 S(p-\frac{q_1}{2}+\frac{q_2}{2})\hat{q}_2 S(p-\frac{q_1}{2}-\frac{q_2}{2})\Big], \tag{36}$$

where $\tau = \beta/\alpha$. First, we make use of the WTI given by Eq. (26) to simplify the product of the free fermion propagators

$$S(p+\frac{q_1}{2}+\frac{q_2}{2})\hat{q}_1 S(p-\frac{q_1}{2}+\frac{q_2}{2})\hat{q}_2 S(p-\frac{q_1}{2}-\frac{q_2}{2})$$
$$= S(p+\frac{q_1}{2}+\frac{q_2}{2}) - S(p-\frac{q_1}{2}+\frac{q_2}{2})$$
$$+S(p+\frac{q_1}{2}+\frac{q_2}{2})\hat{q}_1 S(p-\frac{q_1}{2}-\frac{q_2}{2}). \tag{37}$$

As a result, Eq. (36) assumes the following form

$$\delta G^{(4)}(p)=\int_0^1\frac{d\alpha}{\alpha^{n-2}}\int_0^{1/\alpha}d\tau\int\frac{d^nq_1}{(2\pi)^n}\frac{d^nq_2}{(2\pi)^n}$$
$$\times(2\pi)^n\delta^n(q_1\tau+q_2)D(q_1)$$
$$\times\frac{\partial}{\partial q_1^\lambda}\frac{\partial}{\partial q_2^\lambda}\left[S(p+\frac{q_1}{2}+\frac{q_2}{2})-S(p-\frac{q_1}{2}+\frac{q_2}{2})\right], \tag{38}$$

where we have dropped the last term in Eq. (37) which vanishes upon angular integration. Next, we pull the derivatives to the front of the integral

$$\delta G^{(4)}(p)=\frac{1}{4}\frac{\partial}{\partial p_\lambda}\frac{\partial}{\partial p_\lambda}\int_0^1\frac{d\alpha}{\alpha^{n-2}}\int_0^{1/\alpha}d\tau\int\frac{d^nq_1}{(2\pi)^n}\frac{d^nq_2}{(2\pi)^n}$$
$$\times(2\pi)^n\delta^n(q_1\tau+q_2)D(q_1)\left[S(p+q_1/2+q_2/2)\right.$$
$$\left.+S(p-q_1/2+q_2/2)\right] \tag{39}$$

and carry out the momentum integration over q_2 followed by rescaling of the remaining momentum variable q_1 which yields

$$\delta G^{(4)}(p)=\frac{1}{4}\frac{\partial}{\partial p_\lambda}\frac{\partial}{\partial p_\lambda}\int_0^1\frac{d\alpha}{\alpha^{n-2}}\int_0^{1/\alpha}d\tau\int\frac{d^nq}{(2\pi)^n}D(q)$$
$$\times\left[S(p+q(1-\tau)/2)+S(p-q(1+\tau)/2)\right]$$
$$=\frac{2^n}{N}\frac{\partial}{\partial p_\lambda}\frac{\partial}{\partial p_\lambda}\int_0^1\frac{d\alpha}{\alpha^{n-2}}\int_0^{1/\alpha}d\tau\,\left[(1-\tau)^{1-n}\right.$$
$$\left.+\,(1+\tau)^{1-n}\right]\int\frac{d^nq}{(2\pi)^n}\frac{(\hat p+\hat q)}{\sqrt{q^2}(p+q)^2}$$
$$\approx\frac{4}{\pi^2N}\frac{|p|^{-\epsilon}}{\epsilon}, \tag{40}$$

where we have invoked Eq. (23) to compute the integral

$$\int\frac{d^nq}{(2\pi)^n}\frac{(\hat p+\hat q)}{\sqrt{q^2}(p+q)^2}=\frac{\hat p}{6\pi^2}\frac{|p|^{-\epsilon}}{\epsilon} \tag{41}$$

and used the $n\to3$ asymptotic

$$\int_0^1\frac{d\alpha}{\alpha^{n-2}}\int_0^{1/\alpha}d\tau\left[(1-\tau)^{1-n}+(1+\tau)^{1-n}\right]\simeq-\frac{1}{\epsilon}. \tag{42}$$

Notably, a singular $(1/\epsilon)$ term which is present in the gauge propagator $D^{(4)}_{\mu\nu}(q_1,q_2)$ and the origin of which is discussed in the Appendix gets canceled in Eq. (40).

Combining the four contributions (22), (34), and (40) we finally obtain

$$\frac{1}{4}\mathrm{Tr}\left[\hat p\delta G(p)\right]=\frac{1}{4}\mathrm{Tr}\left[\hat p\left(\delta G^{(1)}(p)+2\delta G^{(2)}(p)+\right.\right.$$
$$\left.\left.\delta G^{(4)}(p)\right)\right]=\frac{32}{3\pi^2N}\frac{|p|^{-\epsilon}}{\epsilon}, \tag{43}$$

which implies that in the momentum space Eq. (2) acquires the form

$$G_{inv}(p)=S(p)+\delta G(p)=\frac{\hat p}{p^2}\left[1-\eta\left(1/\epsilon-\ln|p|\right)\right], \tag{44}$$

thus allowing one to read off the anomalous dimension

$$\eta=-\frac{32}{3\pi^2N}. \tag{45}$$

This result corroborates the earlier calculations performed in the covariant and axial gauges in the framework of both the path-integral approach of Ref.[10] and the direct perturbative expansion of Ref.[13].

III. GAUGE INVARIANT PROPAGATOR WITH SEMI-INFINITE STRINGS

In this section we set out to investigate the dependence of the amplitude (2) on the choice of the contour Γ. Specifically, we consider the contour consisting of two (anti)parallel semi-infinite strings attached to the end points, in which case the source current $J_\mu(z)$ in the line integral (8) is given by the expression

$$J^\mu(z)=n^\mu\int_0^\infty d\alpha\left[\delta^n(z-x-n\alpha)\pm\delta^n(z-y\pm n\alpha)\right], \tag{46}$$

where n^μ is a unit vector in a direction of strings. The upper (lower) signs in (46) correspond to the cases of parallel and antiparallel strings, respectively.

In the former case, despite the fact that the strings do not form a closed contour, the corresponding amplitude remains gauge invariant, as long as all the infinitely remote points can be compactified into a single one. This customary assumption always holds in the perturbative sector of the gauge theory where all the fields vanish at infinity.

The correction to the free fermion propagator has a form

$$\delta G(x-y)=-\frac{1}{2}\int d^nz_1d^nz_2J_\mu(z_1)D_{\mu\nu}(z_1-z_2)J_\nu(z_2)$$
$$\times S(x-y)+\int d^nz_1d^nz_2S(x-z_1)\gamma_\mu S(z_1-y)$$
$$\times D_{\mu\nu}(z_1-z_2)J_\nu(z_2)-\int d^nz_1d^nz_2S(x-z_1)\gamma_\mu$$
$$\times S(z_1-z_2)\gamma_\mu S(z_2-y)D_{\mu\nu}(z_1-z_2). \tag{47}$$

Making use of the amplitude $G(x-y)$ being explicitly gauge invariant, we choose to compute it in the Feynman gauge where the gauge propagator takes a particularly simple form

$$D_{\mu\nu}(x)=\delta_{\mu\nu}\frac{A}{(x^2)^{\frac{n-1}{2}}},\quad A=\frac{4}{N}\frac{\mu^{3-n}\Gamma(\frac{n-1}{2})}{\pi^{\frac{n+1}{2}}}. \tag{48}$$

The last term in Eq. (47) corresponds to the standard fermion self-energy in the Feynman gauge

$$\delta G^{(3)}(x-y) \simeq \frac{4(2-n)\Gamma\left(\frac{n-1}{2}\right)}{N\pi^{\frac{n+1}{2}}(2n-3)}\frac{|x-y|^{3-n}}{3-n}S(x-y). \quad (49)$$

The first term in Eq. (47) containing two source currents is given by the integral

$$\delta G^{(1)}(x-y) = -\frac{1}{2}\int d^n z_1 d^n z_2\, J_\mu(z_1)D_{\mu\nu}(z_1-z_2)J_\nu(z_2)$$
$$= -\frac{A}{2}\int_0^\infty d\alpha\int_0^\infty d\beta\left[\frac{2}{|\alpha-\beta|^{n-1}} - \frac{1}{[(x-y+n(\alpha-\beta))^2]^{\frac{n-1}{2}}}\right.$$
$$\left. - \frac{1}{[(-x+y+n(\alpha-\beta))^2]^{\frac{n-1}{2}}}\right]. \quad (50)$$

The integral over β is convergent for $1<n<2$. However, the remaining integration over α diverges with the upper limit L which we impose as a cut-off.

Rescaling the integration variables $\alpha \to \alpha|x-y|, \beta \to \beta|x-y|$ and introducing the angle θ according to the relation $\cos\theta = (x-y)\cdot n/|x-y|$ we rewrite Eq. (50) as

$$\delta G^{(1)}(x-y) = -\frac{A|x-y|^{3-n}}{2}\int_0^{L/|x-y|} d\alpha\int_0^\infty d\beta\left[\frac{2}{|\alpha-\beta|^{n-1}}\right.$$
$$- \frac{1}{[1+2\cos\theta(\alpha-\beta)+(\alpha-\beta)^2]^{\frac{n-1}{2}}}$$
$$\left. - \frac{1}{[1-2\cos\theta(\alpha-\beta)+(\alpha-\beta)^2]^{\frac{n-1}{2}}}\right]. \quad (51)$$

In the integral over β, we consider separately the intervals from 0 to α and from α to ∞. First we compute the integral from α to ∞ which, upon shifting the integration variable $\beta \to \beta+\alpha$, acquires the form

$$I_1 = \int_{-\infty}^\infty d\tau\left[\frac{1}{(\tau^2)^{\frac{n-1}{2}}} - \frac{1}{[1-2\tau\cos\theta+\tau^2]^{\frac{n-1}{2}}}\right]. \quad (52)$$

After exponentiating the denominators and carrying out the integral over τ one obtains

$$I_1 = \frac{\sqrt{\pi}}{\Gamma(\frac{n-1}{2})}\int_0^\infty ds s^{\frac{n}{2}-2}\left[1-e^{-s\sin^2\theta}\right]$$
$$= -\frac{\sqrt{\pi}\Gamma(\frac{n}{2}-1)}{\Gamma(\frac{n-1}{2})}(\sin^2\theta)^{1-\frac{n}{2}}. \quad (53)$$

In turn, the integral over β from 0 to α takes the form

$$I_2 = \int_0^\alpha \frac{d\beta}{[1+2\cos\theta(\alpha-\beta)+(\alpha-\beta)^2]^{\frac{n-1}{2}}}$$
$$= \alpha\int_0^1 \frac{d\beta}{[1+2\alpha\beta\cos\theta+\alpha^2\beta^2]^{\frac{n-1}{2}}}, \quad (54)$$

where we made a change of variables $\beta \to \alpha-\beta$, followed by rescaling $\beta \to \alpha\beta$.

Next, we represent the last expresion as the difference between the integrals taken from 0 to ∞ and that from 1 to ∞

$$I_2 = \int_0^\infty \frac{d\beta}{[\beta^2+2\beta\cos\theta+1]^{\frac{n-1}{2}}}$$
$$-(\alpha^2+2\alpha\cos\theta+1)^{1-\frac{n}{2}}\int_0^\infty \frac{d\beta}{[\beta^2+2\beta\cos\gamma+1]^{\frac{n-1}{2}}}, \quad (55)$$

where

$$\cos\gamma = \frac{\alpha+\cos\theta}{\sqrt{\alpha^2+2\alpha\cos\theta+1}}. \quad (56)$$

The integrals in Eq. (55) are evaluated with the use of the formula

$$\int_0^\infty \frac{dx}{(x^2\pm 2x\cos\gamma+1)^\rho} = \frac{1}{2}(\sin^2\gamma)^{\frac{1}{2}-\rho}\frac{\sqrt{\pi}\Gamma(\rho-\frac{1}{2})}{\Gamma(\rho)}$$
$$\mp\cos\gamma F\left(1,\rho;\frac{3}{2};\cos^2\gamma\right). \quad (57)$$

Hence the sum of the two integrals reads

$$I_2(\cos\theta)+I_2(-\cos\theta) =$$
$$= (\alpha+\cos\theta)(\alpha^2+2\alpha\cos\theta+1)^{\frac{1-n}{2}}$$
$$\times\ F\left(1,\frac{n-1}{2};\frac{3}{2};\cos^2\gamma\right)+(\cos\theta\to-\cos\theta), \quad (58)$$

which can be further transformed by using the relation between the hypergeometric functions of complementary arguments

$$F(a,b;c;z) =$$
$$= \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}F(a,b;a+b-c+1;1-z)$$
$$+\ (1-z)^{c-a-b}\frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}$$
$$\times\ F(c-a,c-b;c-a-b+1;1-z), \quad (59)$$

thus resulting in the expression

$$I_2(\cos\theta)+I_2(-\cos\theta) = \frac{\sqrt{\pi}\Gamma(\frac{n}{2}-1)}{2\Gamma(\frac{n-1}{2})}(\sin^2\theta)^{1-\frac{n}{2}}$$
$$\times\left(\frac{\alpha+\cos\theta}{|\alpha+\cos\theta|}+\frac{\alpha-\cos\theta}{|\alpha-\cos\theta|}\right)$$
$$+\frac{1}{2-n}\left[(\alpha+\cos\theta)(\alpha^2+2\alpha\cos\theta+1)^{\frac{1-n}{2}}\right.$$
$$\left.\times\ F\left(1,\frac{n-1}{2};\frac{n}{2};\sin^2\gamma\right)+(\cos\theta\to-\cos\theta)\right]. \quad (60)$$

Invoking the formula for the derivatives of the hypergeometric function

$$F(a+n,b;c;z)=\frac{z^{1-a}}{(a)_n}\frac{d^n}{dz^n}[z^{a+n-1}F(a,b;c;z)]\,, \qquad (61)$$

we can rewrite the expression inside the square brackets in Eq. (58) as a total derivative. Then the integration over α becomes trivial and we finally get the expression for the double-source term

$$\delta G^{(1)}(x-y)=\frac{A\sqrt{\pi}\Gamma(\frac{n}{2}-1)}{\Gamma(\frac{n-1}{2})}[L-\frac{1}{2}|x-y|\cos\theta]$$
$$\times|x-y|^{2-n}(\sin^2\theta)^{1-\frac{n}{2}}-\frac{A|x-y|^{3-n}}{(2-n)(3-n)}$$
$$\times F\left(1,\frac{n-3}{2};\frac{n}{2};\sin^2\theta\right), \qquad (62)$$

which behaves as

$$\delta G^{(1)}(x-y)\simeq\frac{4}{N\pi^2}\frac{(\mu|x-y|)^{3-n}}{3-n}\,, \qquad (63)$$

near $n=3$, independent of the cutoff L.

The second term in Eq. (45) containing one insertion of the source current is given by the integral

$$\delta G^{(2)}(x-y)=\int_0^\infty d\alpha\int d^nz\,S(x-z)\hat{n}S(z-y)$$
$$\times[D(z-y-\alpha n)-D(z-x-\alpha n)]. \qquad (64)$$

This expression can be readily computed in the momentum space where it reads as

$$\delta G^{(2)}(x-y)=-\int\frac{d^np}{(2\pi)^n}e^{-ip(x-y)}\int_0^\infty d\alpha\int\frac{d^nq}{(2\pi)^n}$$
$$\times e^{i\alpha qn}[S(p)\hat{n}S(p-q)-S(p+q)\hat{n}S(p)]D(q). \qquad (65)$$

First, we consider the integral

$$J=\int\frac{d^nq}{(2\pi)^n}e^{i\alpha qn}S(p-q)D(q)$$
$$=\frac{8}{N}\int\frac{d^nq}{(2\pi)^n}e^{i\alpha qn}\frac{\hat{p}-\hat{q}}{(p-q)^2}\frac{1}{\sqrt{q^2}}\,, \qquad (66)$$

which, upon exponentiating the denominators and integrating over q, takes the form

$$J=\frac{8}{N\sqrt{\pi}(4\pi)^{n/2}}\int_0^\infty ds\int_0^\infty\frac{dt}{\sqrt{t}}\frac{1}{(s+t)^{\frac{n}{2}+1}}$$
$$\times\ e^{-p^2\frac{st}{s+t}+i\alpha\frac{s}{s+t}pn-\frac{\alpha^2}{4(s+t)}}\left(\hat{p}t-\frac{i\alpha\hat{n}}{2}\right). \qquad (67)$$

Thus Eq. (63) can be written as

$$\delta G^{(2)}(x-y)=-\frac{8}{N\sqrt{\pi}(4\pi)^{n/2}}\int\frac{d^np}{(2\pi)^n}e^{-ip(x-y)}$$
$$\times\int_0^\infty d\alpha\int_0^\infty ds\int_0^\infty\frac{dt}{\sqrt{t}}\frac{1}{(s+t)^{\frac{n}{2}+1}}e^{-p^2\frac{st}{s+t}-\frac{\alpha^2}{4(s+t)}}\Big[e^{i\alpha\frac{s}{s+t}pn}$$
$$\times\,S(p)\hat{n}\left(\hat{p}t-\frac{i\alpha\hat{n}}{2}\right)-e^{-i\alpha\frac{s}{s+t}pn}\left(\hat{p}t+\frac{i\alpha\hat{n}}{2}\right)\hat{n}S(p)\Big]. \qquad (68)$$

After inserting into the integrand the identity $\int_0^\infty d\rho\delta(\rho-s-t)=1$ and rescaling the variables $s\to s\rho, t\to t\rho$ one can readily perform the integration over s.

The integration over ρ results in the table integral

$$\int_0^\infty dx x^{\alpha-1}e^{-px-q/x}=2\left(\frac{q}{p}\right)^{\alpha/2}K_\alpha(2\sqrt{pq}), \qquad (69)$$

thus yielding

$$\delta G^{(2)}(x-y)=-\frac{32i}{N\sqrt{\pi}(4\pi)^{n/2}}\int\frac{d^np}{(2\pi)^n}e^{-ip(x-y)}$$
$$\times(2|p|)^{\frac{n-3}{2}}\int_0^\infty d\alpha\alpha^{\frac{3-n}{2}}\int_0^1 dt[t(1-t)]^{\frac{n-3}{4}}\Big[\sqrt{1-t}$$
$$\times\,\sin(\alpha tpn)S(p)\hat{n}\hat{p}\,K_{\frac{3-n}{2}}(\alpha|p|\sqrt{t(1-t)})$$
$$-\,|p|S(p)\sqrt{t}\cos(\alpha tpn)K_{\frac{1-n}{2}}(\alpha|p|\sqrt{t(1-t)})\Big]. \qquad (70)$$

The remaining integrals over α are given by the formulas 6.699.3 and 6.699.4 from the Integral Tables[15].

Thus, we arrive at the formula

$$\delta G^{(2)}(x-y)=-\frac{i2^{5-n}}{N\pi^{\frac{n+1}{2}}}\int\frac{d^np}{(2\pi)^n}e^{-ip(x-y)}|p|^{n-4}$$
$$\times\int_0^1 dt t^{\frac{n-3}{2}}(1-t)^{\frac{n-4}{2}}\left[S(p)\hat{n}\hat{p}\cdot\frac{pn}{|p|}\Gamma\left(\frac{5-n}{2}\right)\right.$$
$$\times\,F\left(1,\frac{5-n}{2};\frac{3}{2};-\frac{t(pn)^2}{(1-t)p^2}\right)-\frac{1}{2}|p|S(p)$$
$$\left.\times\,\Gamma\left(\frac{3-n}{2}\right)F\left(1,\frac{3-n}{2};\frac{1}{2};-\frac{t(pn)^2}{(1-t)p^2}\right)\right], \qquad (71)$$

where the integration over t can be performed by changing the variable $t=u/(1+u)$ and comparing the result with the integral representation for the hypergeometric function ${}_3F_2$ of a certain argument.

However, one can notice that at $n\to3$ the main contribution stems from the second term in the square brackets

$$\delta G^{(2)}(x-y)\simeq\frac{2\Gamma\left(n-\frac{3}{2}\right)\Gamma\left(\frac{3-n}{2}\right)\mu^{3-n}}{N\pi^{n+\frac{1}{2}}\Gamma\left(\frac{5-n}{2}\right)}\frac{\hat{x}-\hat{y}}{[(x-y)^2]^{n-\frac{3}{2}}}$$
$$=\frac{8\Gamma\left(n-\frac{3}{2}\right)}{N\pi^{\frac{n+1}{2}}\Gamma\left(\frac{n}{2}\right)}\frac{(\mu|x-y|)^{3-n}}{3-n}S(x-y), \qquad (72)$$

where we restored the dependence on the dimensionful parameter μ and also used

$$S(x) = \frac{\Gamma(n/2)}{2\pi^{n/2}} \frac{\hat{x}}{(x^2)^{n/2}}. \tag{73}$$

Combining Eqs. (47), (61), and (70) together, we find the overall correction to the fermion propagator

$$G(x-y) \approx \left[1 + \frac{32}{3\pi^2 N} \frac{(\mu|x-y|)^{3-n}}{3-n}\right] S(x-y), \tag{74}$$

from which one can read off the anomalous dimension.

Remarkably, the latter appears to be still given by Eq. (45), as in the case of the original "short-cut" contour studied in the previous Section.

Furthermore, a similar calculation shows that the negative anomalous dimension Eq. (45) also pertains to the case of the parallel strings which corresponds to choosing the upper sign in Eq. (46).

Taken at their face value, these observations suggest that the gauge invariant amplitude Eq. (2) may even be largely independent of the choice of the contour Γ.

IV. SUMMARY

In this work, we carried out a direct calculation of the previously conjectured form of the physical electron propagator in such effective QED-like models as the theory of the pseudogap phase of the cuprates. In contrast to the earlier work, we performed our calculations in the reliable radial gauge and confirmed the result (45) obtained in Refs.[10,13].

In the course of our analysis, we also investigated the dependence of the amplitude (2) on the choice of the contour Γ by considering the case of two (anti)parallel semi-infinite strings attached to the end points. The corresponding gauge invariant amplitude is given by Eq. (2) with the current Eq. (46) entering the line integral (8). Remarkably, the algebraic behavior (1) controlled by the same negative anomalous exponent (45) appears to be valid for these functions as well.

In addition to the possible dependence on the choice of the contour Γ (or a lack thereof), the anomalous dimension may strongly depend on the massless fermion amplitude in question. For instance, when computed in one of the covariant gauges, the amplitude

$$G_\xi(x-y) = \frac{\langle 0|\psi(x) \exp\left[i(\xi-1)\int_y^x dz^\mu\, A_\mu(z)\right]\bar{\psi}(y)|0\rangle}{\langle 0|\exp\left[i\xi\int_y^x dz^\mu\, A_\mu(z)\right]|0\rangle} \tag{75}$$

exhibits a positive anomalous dimension

$$\eta_\xi = \frac{16}{3\pi^2 N}(3\xi - 2) \tag{76}$$

for any $\xi > 2/3$[13], including the case of $\xi = 1$ which has been claimed[11] to provide an identical representation of the original function $G_0(x)$ given by Eq. (2).

However, for any $\xi \neq 0$ the amplitude $G_\xi(x)$ given by (75) is not truly gauge-invariant, and, in particular, its anomalous dimension computed in a non-covariant gauge may differ from Eq. (76). (for an extended discussion of this subtle point, see[13]). For instance, when computed in the radial gauge applied in this paper the anomalous dimension of the function $G_\xi(x)$ turns out to be independent of ξ and is given by Eq. (45).

This makes it clearly impossible to substitute any of the surrogate amplitudes $G_\xi(x)$ with $\xi > 2/3$ (e.g., $G_1(x)$, as in Ref.[11]) for the original one, $G_0(x)$, which is the only truly gauge-invariant member of the family of functions (75).

Evidently, the negative anomalous dimension manifested by the function (2) contradicts the anticipated behavior of a viable candidate to the role of the physical electron propagator, since in all of the previously discussed effective QED_3-like models the repulsive electron interactions are expected to result in further suppression, rather than enhancement, of any amplitude describing propagation of physical electrons.

In particular, the algebraic decay of the gauge-invariant fermion amplitude (1) would only result in the sought-after Luttinger-like (stronger-than-linear) vanishing density of states $\nu(\epsilon) \sim |\epsilon|^{1+\eta}$ if η were positive. By the same token, a pseudogap theory can only reconcile with the experimentally established absence of well-defined nodal quasiparticles if a branch-cut singularity of the electron propagator that occurs at $p_\mu^2 = 0$ appears to be weaker (not stronger!) than a simple pole.

We defer a further discussion of the construction of the physical electron propagator until future work (see, however, Refs.[10,13] for an alternate form which demonstrates a faster-than-algebraic decay, thus further diminishing the chances that the conjecture about the Luttinger-like behavior in QED_3 may still be "right, albeit for a wrong reason").

Instead, we suggest that the negative anomalous dimension (45) of the heuristically chosen gauge-invariant amplitude (2) may pertain not so much to the physical electron propagator *per se*, but rather to the vertex corrections which also control the behavior of various gauge-invariant two-particle amplitudes ("susceptibilities").

In this regard, we quote the earlier result of Ref.[17] obtained for the susceptibility associated with the four-fermion scalar vertex

$$< 0|\overline{\psi}(x)\psi(x)\overline{\psi}(y)\psi(y)|0 > \propto \frac{1}{|x-y|^{4-(64/3\pi^2 N)}} \tag{77}$$

which features a negative anomalous dimension 2η. In the context of the QED_3 theory of the pseudogap phase of the cuprates, the formula (77) describes the divergence of the staggered spin susceptibility at the antiferromagnetic ordering vector $\vec{Q} = (\pi, \pi)$[18].

We emphasize that one encounters the above problem with the unphysical (slower than $\propto 1/x^2$) decay of the amplitude (2) only in the massless case, while for a finite fermion mass this function decays as $\propto e^{-m|x|}$.

In the case of the QED_3 theory of the pseudogap phase, it has been argued that one may indeed expect a dynamical mass generation corresponding to the intrinsic instability towards a spin and/or charge density wave ordering[8,11].

The question remains, though, as to whether or not the chiral symmetry breaking can at all occur for the physical number of fermion flavors ($N = 2$). Even in the fully Lorentz-invariant situation there exist some analytical[19] and numerical[20] results which suggest the upper bound $N_{cr} < 2$ for the critical number of flavors below which the chiral symmetry gets broken.

In the (non-Lorentz-invariant) QED_3 theory of the pseudogap phase of Refs.[4–8], the role of the strong spatial anisotropy of the quasiparticle dispersion and, in particular, its effect on a possible universality (or a lack thereof) of the critical value of N_{cr} still remain to be ascertained (see[21] for a discussion of the weakly anisotropic case).

It is worth mentioning, however, that in the extreme non-Lorentz-invariant limit of the QED_3-like theory describing the problem of layered graphite the estimated value of N_{cr} was found to be even lower than in the original Lorentz-invariant case[22].

We conclude by stressing that the problem of constructing the true physical electron propagator in the effective massless QED-like theories still remains unresolved. Nevertheless, our calculation confirms once and for all that the naive ansatz (2) is not up to the job, thereby eliminating the current basis for the theoretical predictions of the Luttinger-like behavior in the underdoped cuprates[5–7].

It is, however, conceivable that, while being unrelated to the actual behavior of the electron propagator, the negative anomalous dimension (45) manifests the same properties of the gauge invariant vertex corrections as those exhibited by the physically relevant two-fermion amplitudes such as Eq. (77).

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APPENDIX A: FOCK-SCHWINGER PHOTON PROPAGATOR

In this Appendix we demonstrate that the line integral in Eq. (2) vanishes in the so-called radial or Fock-Schwinger (FS) gauge. We also derive the expression for the photon propagator in this gauge.

The FS gauge is defined as

$$(x-x_0)^\mu A_\mu(x,x_0) = 0. \tag{A1}$$

In contrast to such widely used gauges as the Landau $\partial^\mu A_\mu(x) = 0$, the Coulomb $\partial_i A_i(x) = 0 (i = 1,2)$ and the axial $n_\mu A_\mu(x) = 0$ ones, the FS gauge may break the translational invariance because of the presence of a fixed point x_0. However, an important advantage of the Fock gauge is the explicit relation between the potential $A_\mu(x,x_0)$ and the field strength $F_{\mu\nu}$

$$A_\mu(x,x_0) = \int_0^1 d\alpha\, \alpha (x-x_0)^\nu F_{\nu\mu}(\alpha(x-x_0)+x_0, x_0). \tag{A2}$$

In order to derive Eq. (A2) we differentiate Eq. (A1)

$$A_\mu(x,x_0) + (x-x_0)^\nu \partial_\mu A_\nu(x,x_0) = 0, \tag{A3}$$

and then use $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, to write

$$A_\mu(x,x_0) + (x-x_0)^\nu (F_{\mu\nu}(x,x_0) + \partial_\nu A_\mu(x,x_0)) = 0. \tag{A4}$$

Upon changing the variable $x \to \alpha(x-x_0)+x_0$ the last expression turns into

$$\begin{aligned}&\frac{d}{d\alpha}[\alpha A_\mu(\alpha(x-x_0)+x_0,x_0)]\\ &= \alpha(x-x_0)^\nu F_{\nu\mu}(\alpha(x-x_0)+x_0,x_0).\end{aligned} \tag{A5}$$

Integrating over α and using the boundary condition $A_\mu(x_0,x_0) = 0$ (see, Eq. (A3)) which assumes the regularity of $A_\mu(x,x_0)$ at $x = x_0$, we arrive at Eq. (A2). Note that the boundary condition $A_\mu(x_0,x_0) = 0$ is essential for eliminating a residual gauge freedom which remains even after imposing the gauge condition (A1).

Indeed, in addition to the solution (A2) Eq. (A1) can be satisfied by any function

$$A^0_\mu(x,x_0) = \partial^x_\mu f(x-x_0), \tag{A6}$$

where f is an arbitrary homogeneous function of $x - x_0$ of zero degree. Any such function would necessarily be singular at $x = x_0$, though. Hence, the regularity condition at $x - x_0$ can be used to fix the residual gauge freedom in (A1).

Under the translation $U_a^{-1} F_{\mu\nu}(x) U_a = F_{\mu\nu}(x-a)$ the solution (A2) transforms as

$$U_a^{-1} A_\mu(x, x_0) U_a = A'_\mu(x, x_0) = A_\mu(x-a, x_0-a). \quad (A7)$$

When expressed in terms of the center mass $X = (x+y)/2$ and the relative $\bar{x} = x - y$ coordinates the line integral in Eq. (2) takes the following form

$$I(\bar{x}, X; x_0) = \int_y^x dz_\mu A^\mu(z)$$
$$= (x-y)^\mu \int_0^1 d\alpha A_\mu(\alpha(x-y)+y, x_0)$$
$$= \bar{x}^\mu \int_{-1/2}^{1/2} d\alpha A_\mu(\alpha\bar{x}+X, x_0). \quad (A8)$$

Under translations, Eq. (A8) transforms according to the rule: $I(\bar{x}, X; x_0) = I(\bar{x}, X-a; x_0-a)$.

We can further restrict the gauge condition (A1) by choosing the fixed point x_0 at the center of mass, *i.e.*, $x_0 = X$

$$(x-X)^\mu A_\mu(x, X) = 0 \quad (A9)$$

(hereafter, we simplify the notation $A_\mu(x, x_0 = X) \equiv A_\mu(x)$).

It can be readily seen that in the gauge (A9) the line integral vanishes, *i.e.*, $I(\bar{x}, X; X) = 0$. Indeed, from Eqs. (A8) and (A2) we obtain

$$I(\bar{x}, X; X) = \bar{x}^\mu \int_{-1/2}^{1/2} d\alpha A_\mu(\alpha\bar{x}+X)$$
$$= \bar{x}^\mu \bar{x}^\nu \int_{-1/2}^{1/2} d\alpha \int_0^1 d\beta\beta F_{\nu\mu}(\alpha\beta\bar{x}+X) = 0 \quad (A10)$$

due to the antisymmetry of $F_{\nu\mu}$.

Furthermore, performing a translation with $a = X$ we can cast the gauge condition (A9) in the form

$$(x-X)^\mu A_\mu(x-X) = 0, \quad (A11)$$

which is identical to

$$x^\mu A_\mu(x) = 0. \quad (A12)$$

Next, we derive the photon propagator in the gauge (A12) where Eq. (A2) reads as

$$A_\mu(x) = \int_0^1 d\alpha\, \alpha x^\nu F_{\nu\mu}(\alpha x). \quad (A13)$$

Thus, we find

$$D^{FS}_{\mu\nu}(x, y) = \langle 0|TA_\mu(x)A_\nu(y)|0\rangle$$
$$= \int_0^1 d\alpha d\beta\, \alpha\beta x^\sigma y^\rho \langle 0|TF_{\sigma\mu}(\alpha x)F_{\rho\nu}(\beta y)|0\rangle. \quad (A14)$$

Since the field strength $F_{\mu\nu}$ is a gauge invariant quantity, the correlator $\langle 0|TF_{\sigma\mu}(x)F_{\rho\nu}(y)|0\rangle$ can be calculated in any gauge, including, e.g., the Feynman one where it becomes

$$x^\sigma y^\rho \langle 0|TF_{\sigma\mu}(x)F_{\rho\nu}(y)|0\rangle = x^\sigma y^\rho \left(\delta_{\mu\nu}\partial^x_\sigma\partial^y_\rho - \delta_{\mu\rho}\partial^x_\sigma\partial^y_\nu - \delta_{\sigma\nu}\partial^x_\mu\partial^y_\rho + \delta_{\sigma\rho}\partial^x_\mu\partial^y_\nu\right) D(x-y)$$
$$\equiv H_{\mu\nu}(x, y) D(x-y). \quad (A15)$$

Here $D(x)$ is the photon propagator in the Feynman gauge

$$D(x) = \frac{A}{(x^2)^{\frac{n-1}{2}}}, \quad A = \frac{4}{N} \frac{\mu^{3-n}\Gamma(\frac{n-1}{2})}{\pi^{\frac{n+1}{2}}}. \quad (A16)$$

With the use of the relation $x_\mu\partial_\mu = |x|\partial_{|x|}$ the operator $H_{\mu\nu}(x, y)$ can be written in the form

$$H_{\mu\nu}(x, y) = \delta_{\mu\nu}\partial_{|x|}\partial_{|y|}|x||y| - \partial^x_\mu x_\nu \partial_{|y|}|y| - \partial^y_\nu y_\mu \partial_{|x|}|x| + \partial^x_\mu\partial^y_\nu x \cdot y. \quad (A17)$$

Now, making use of the identity

$$\partial_{|x|} \int_0^1 d\alpha\, |x| f(\alpha x) = \partial_{|x|} \int_0^1 d\alpha\, |x| f(\alpha|x|\hat{x})$$
$$= \partial_{|x|} \int_0^{|x|} d\beta f(\beta\hat{x}) = f(x), \quad (A18)$$

where $\hat{x} = x/|x|$ is the unit vector, we obtain

$$D_{\mu\nu}(x, y) = \langle 0|TA_\mu(x)A_\nu(y)|0\rangle$$
$$= \int_0^1 d\alpha d\beta\, H_{\mu\nu}(\alpha x, \beta y) D(\alpha x - \beta y)$$
$$= H_{\mu\nu}(x, y) \int_0^1 d\alpha d\beta D((\alpha x - \beta y)^2) = \delta_{\mu\nu} D(x-y)$$
$$- \int_0^1 d\alpha\, \partial^x_\mu x_\nu D(\alpha x - y) - \int_0^1 d\beta\, \partial^y_\nu y_\mu D(x - \beta y)$$
$$+ \int_0^1 d\alpha d\beta\, \partial^x_\mu \partial^y_\nu x \cdot y D(\alpha x - \beta y) \quad (A19)$$

(cf. Refs.[23,24]).

While the first term in (A19) depends solely on $\bar{x}$, the others have a more complicated dependence, hence the photon propagator computed in an arbitrary FS gauge is

not necessarily translationally invariant. Moreover, the last term in Eq. (A19) displays a divergence at $n = 3$ which forces one to use the method of dimensional regularization when computing various quantities. As pointed out in Ref.[24] (also, see Eq. (40)), the divergence of the free FS gauge propagator at $n = 3$ dimensions is, in fact, necessary for obtaining the correct results.

Finally, in order to return to the gauge (A9) we replace $x \to x - X, y \to y - X$ in (A19), thus obtaining the FS gauge propagator $D^{FS}_{\mu\nu}(x, y)$ as a sum of the four terms (here the arguments x and y are unrelated to the end points in the line integral)

$$D^{(1)}_{\mu\nu}(x, y) \equiv \delta_{\mu\nu} D(x - y), \tag{A20}$$

$$D^{(2)}_{\mu\nu}(x, y) \equiv -\int_0^1 d\alpha \partial^x_\mu (x - X)_\nu D(\alpha x - y + (1 - \alpha)X), \tag{A21}$$

$$D^{(3)}_{\mu\nu}(x, y) \equiv -\int_0^1 d\beta \partial^y_\nu (y - X)_\mu D(x - \beta y - (1 - \beta)X), \tag{A22}$$

$$D^{(4)}_{\mu\nu}(x, y) \equiv \int_0^1 d\alpha d\beta \partial^x_\mu \partial^y_\nu (x - X) \cdot (y - X) D(\alpha x - \beta y - (\alpha - \beta)X). \tag{A23}$$

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